ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

AUTOMORPHISM GROUPS OF 3–NODAL RATIONAL CURVES

R.F. LAX

While much is known about automorphisms of smooth projective curves, relatively few results apparently exist concerning automorphisms of singular curves. In this note, we consider the simplest type of irreducible singular curves– rational curves with only nodes as singularites. We give a bound for the order of the automorphism group of such a curve and we determine which groups occur as automorphism groups of 2–nodal and 3–nodal rational curves. We will work over the complex numbers and we will let $\mathbf{P}$ denote the complex projective line, which we will also view as the Riemann sphere. By a "curve", we will mean a reduced and irreducible algebraic variety of (complex) dimension one.

A rational nodal curve of arithmetic genus g (i.e., with g nodes) is isomorphic to the quotient of $\mathbf{P}$ obtained by identifying g pairs of distinct points. If a node Q on such a curve is formed by identifying the points a and b of $\mathbf{P}$, then the local ring of Q is

$$O_Q = \{f \in O_a \bigcap O_b : f(a) = f(b)\}$$

(cf.[4]). Let

$$X = (a_1, b_1; \dots ; a_g, b_g)$$

denote the rational nodal curve of arithmetic genus g obtained by identifying the points a_i and b_i of $\mathbf{P}$ for $i = 1, \dots g$ and let $\pi : \mathbf{P} \to X$ denote the quotient map. We will assume throughout that $g \geq 2$. Let $\operatorname{Aut}(X)$ denote the automorphism group of X and let S_n denote the symmetric group of degree n.

PROPOSITION 1. $\operatorname{Aut}(X)$ *is isomorphic to the group of all automorphisms* ϕ *of* $\mathbf{P}$ *which satisfy the following property:*

(*) *There exists* $\sigma_\phi \in S_g$ *such that* $\phi(\{a_i, b_i\}) = \{a_{\sigma_\phi(i)}, b_{\sigma_\phi(i)}\}$ *for* $i = 1, \dots, g$.

Received by the editors on November 12, 1985.