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UNIQUE SOLVABILITY OF AN AGE-STRUCTURED POPULATION MODEL WITH CANNIBALISM

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ABSTRACT. A modification of the McKendrick–Von Foerster population balance equation is used to model populations in which: (i) no individual lives past age $a = L$ and (ii) young individuals, $0 < a < L^* < L$, are cannibalized by the older, $L^* < a < L$ individuals. The balance equation is formulated as an equivalent integral equation and the contraction mapping principle is used to establish the unique solvability. The existence and stability of equilibrium solutions are considered.

1. Introduction. Let $\rho = \rho(t, a)$ denote the population density of individuals of age a at time t and let

$$D\rho = \lim_{h \to 0} \frac{\rho(t+h, a+h) - \rho(t,a)}{h}.$$

if ρ is continuously differentiable, then $D\rho = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial a}$. The McKendrick-Von Foerster population balance equation, McKendrick 1926, Von Foerster 1959, states

$$D\rho = -\gamma\rho$$

where γ is a nonnegative quantity, often called the death modulus, which determines the rate of removal of individuals from the population due to death. As noted by Gurtin and MacCamy, 1974, the death modulus, γ, may depend on ρ, total population density, age, and time, etc.

The equations considered in this paper are based on the above population balance equation and were motivated by a desire to model populations in which

(i) individuals live to a maximum age of $a = L$;

(ii) young individuals, $0 < a < L^* < L$ are preyed upon (cannibalized) by the older individuals, $L^* < a < L$.

The conditions (i) and (ii) are idealizations of those exhibited by

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