ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

ABSOLUTELY CONTINUOUS SPECTRA OF PERTURBED PERIODIC HAMILTONIAN SYSTEMS

D.B. HINTON AND J.K. SHAW

ABSTRACT. This paper compares the spectrum of the Hamiltonian system $J\vec{y}' = (\lambda R(x) + Q(x))\vec{y}, -\infty < x < \infty$, with periodic coefficient matrices $R(x)$ and $Q(x)$, to that of a perturbed system $J\vec{y}' = (\lambda R(x) + Q(x) + P(x))\vec{y}$, where $P \in L^1_{\mathbf{R}}(-\infty, \infty)$. We show that the perturbation can introduce at most eigenvalues into the gaps between the endpoints of the stability intervals of the periodic system. We prove that the spectral function is continuously differentiable across the continuous spectrum. Further, it follows from the results here that the essential spectrum, the absolutely continuous spectrum and the singular continuous spectrum are invariant.

1. Introduction. We will study the 2×2 Hamiltonian system

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\vec{y}' = \left(\lambda \begin{pmatrix} r_1(x) & r_{12}(x) \\ r_{12}(x) & r_2(x) \end{pmatrix} + \begin{pmatrix} q_1(x) & q_{12}(x) \\ q_{12}(x) & q_2(x) \end{pmatrix}\right)\vec{y}', \quad -\infty < x < \infty, \tag{1.1}$$

which will be assumed to have real and piecewise continuous coefficient matrices which are periodic. We shall write $\vec{y}(x) = \binom{y(x)}{\hat{y}(x)}$ for a solution of (1.1), but otherwise our notation agrees with that of **[8]** and **[24]**. If the coefficients have period T, then $\vec{z}(t) = \vec{y}(tT)$ satisfies an equation of the form (1.1) with coefficients of period 1. Thus we will assume, without loss of generality, that $T = 1$.

Our objective is to examine spectral properties of operators associated with (1.1) as compared with those of operators arising from the

---

A.M.S. (1980) Subject Classification: primary 34B20, 34B25; secondary 47E05.
Key words and phrases: Hamiltonian system, Dirac system, periodic coefficients, spectrum, perturbed spectrum, Hill's equation.
Received by the editors on May 28, 1985 and in revised form on February 3, 1986.