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COMPLETELY MONOTONIC FUNCTIONS OF THE FORM $s^{-b}(s^2+1)^{-a}$

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ABSTRACT. The function $s^{-b}(s^2 + 1)^{-a}$ is shown to be completely monotonic for $b \geq 2a \geq 0$, for $b \geq a \geq 1$. or for $0 \leq a \leq 1$, $b \geq 1$. Moreover this function is proven not to be completely monotonic for $0 \leq b < a$, nor for $a = b$, $0 < a < 1$. This proves some conjectures of Askey [**1**], and extends some of the results of [**2**], [**3**], and [**4**].

1. Introduction. In recent years Askey, Gasper, Ismail, and others have looked into the problem of determining the nonnegativity of the Bessel function integrals $\int_0^t (t-s)^c s^d J_\nu(s)ds$, as well as some ${}_1F_2'$s. See [**2,3**]. This is related to the complete monotonicity of $s^{-a}(s^2+1)^{-b}$ as we shall see in this article.

The definition of complete monotonicity used in this paper is:

DEFINITION. A function $f(s)$ is completely monotonic (C.M.) if

$$(-)^n f^{(n)}(s) \geq 0, s > 0, n = 0, 1, 2, \cdots.$$

The main result we will need is the Hausdorff–Bernstein–Widder theorem [**8**].

THEOREM A. *$f(s)$ is completely monotonic if and only if it is the Laplace Transform of a positive measure on* $(0, \infty)$.

Accordingly, we will make the following definitions.

DEFINITION. Let $\mathcal{L}$ denote the Laplace transform operator and let $\mathcal{L}^{-1}$ denote its inverse. We define:

$$S_{a,b}(t) = \mathcal{L}^{-1}(s^{-a}(s^2+1)^{-b}). \tag{1.1}$$

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