ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

DISCRETE CUBIC SPLINE INTERPOLATION OVER A NONUNIFORM MESH

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1. Introduction. Let us consider a mesh P on $[a, b]$ which is defined by

$$P : a = x_o < x_1 < \cdots < x_n = b.$$

For $i = 1, 2, \cdots, n$, p_i shall denote the length of the mesh interval $[x_{i-1}, x_i]$. Let $p = \max_{1 \le i \le n} p_i$ and $p' = \min_{1 \le i \le n} p_i$. P is said to be a uniform mesh if p_i is a constant for all i. Throughout, h will represent a given positive real number. Consider a real function $s(x, h)$ defined over $[a, b]$ which is such that its restriction s_i on $[x_{i-1}, x_i]$ is a polynomial of degree 3 or less for $i = 1, 2, \cdots, n$. Then $s(x, h)$ defines a discrete cubic spline if

$$(s_{i+1} - s_i)(x_i + jh) = 0, j = -1, 0, 1; i = 1, 2, \cdots, n-1. \tag{1.1}$$

Discrete splines have been introduced by Mangasarian and Schumaker [**5**] in connection with certain studies of minimization problems involving differences. Discrete cubic splines which interpolate given functional values at one point lying in each mesh interval of a uniform mesh have been studied in [**1**]. The case in which these points of interpolation coincide with the mesh points of a nonuniform mesh was studied earlier by Lyche [**3**], [**4**]. The object of the present paper is to study the existence, uniqueness and convergence properties of a discrete cubic spline interpolant of a nonuniform mesh which takes prescribed values at one point of each mesh interval. In comparison to the results proved in [**1**], the use of nonuniform mesh in our results permits a wider choice for the points of interpolation. This will be demonstrated by employing a certain nonuniform geometric mesh. The results obtained in this paper include in particular some earlier results due to Lyche [**4**] and Dikshit and Powar [**1**]. For the corresponding results on continuous

Received by the editors on September 21, 1984, and in revised form on January 23, 1986.
This research was supported by the Grant No. 10/4/115 of IRSO, Dept. of Space, Govt. of India.