ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

RULED HYPERSURFACES OF EUCLIDEAN SPACE

K. ABE AND D.E. BLAIR

In [**2**] J.R. Vanstone and one of the authors studied a minimal hypersurface of Euclidean space E^{n+1} which admits a foliation by Euclidean $(n-1)$–planes. Such a hypersurface was shown to be either totally geodesic or the product $M^2 \times E^{n-2}$ where M^2 is the standard helicoid in E^3. In this paper we are interested in this problem, not as a minimality one, but as a cylindricity problem and in the question of whether or not the mere existence of an $(n-1)$–plane through every point implies that the surface is foliated. Our basic assumption is that for a hypersurface M immersed in E^{n+1} we have the following condition.

CONDITION (*). *Through each point $x \in M$, there exists an entire $(n-1)$–plane contained in M.*

We shall show that for a surface M in E^3, this implies that the surface is ruled (i.e., foliated by lines), but note that the lines of the ruling need not be the lines hypothesized. For example consider a doubly ruled surface (hyperboloid of one sheet, hyperbolic paraboloid or plane) and for the lines of condition (*) make a random choice between the two rulings at each point. In general if the hypersurface M is not foliated by the given $(n-1)$–planes, we have points where these planes intersect. Our main result is to show that in a neighborhood of such a point $(n-2)$–dimensions break away and we have a product structure of an open set in E^{n-2} and a piece of a surface in E^3. If these intersections are dense, M is the product of E^{n-2} and a doubly ruled surface. Finally we show by example that M may be foliated by $(n-1)$–planes but not have a product structure with E^{n-2} as a factor; in particular the complement of the relative null distribution is not integrable.

Let $\overline{\nabla}$ denote the standard connection on E^{n+1} and ∇ the induced connection on M; the second fundamental form α of the immersion is

Received by the editors on June 10, 1985.