ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

SOURCES, SINKS AND SADDLES FOR EXPANSIVE HOMEOMORPHISMS WITH CANONICAL COORDINATES

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ABSTRACT. We study sources, sinks and saddles for expansive homeomorphisms of compact metric spaces which have canonical coordinates. If the phase space is connected and locally connected then each point is a saddle. We show by example that local connectedness is a necessary hypothesis.

1. Introduction. Canonical coordinates were introduced by R. Bowen [**1**]. (see also S. Smale [**9**].) He used expansive homeomorphisms having canonical coordinates to study Axiom A diffeomorphisms [**1,2,3,4**]. This notion was a fruitful one for ergodic theory [**3,5,8**], entropy calculations [**1,5**] and topological dynamics [**4,7**].

Since canonical coordinates "move around with a point" one may extend certain notions which are valid for fixed or periodic points to this setting. We generalize the notions of source, sink and saddle to any point in the phase space of an expansive homeomorphism which has canonical coordinates.

2. Canonical Cordinates. Let f be an expansive homeomorphism of the compact metric (d) space X. Fix an expansive constant $c > 0$ for f. We now define canonical coordinates and related concepts and collect some useful facts, mostly without (the easy) proofs.

DEFINITION 2.1. We define the local stable set of f at $x \in X$ for $\delta \geq 0$ as follows.

$$W^s(x,\delta) = W^s(x,\delta,f) = \{y : d[f^n(x)],[f^n(y)] \leq \delta \text{ for } n \geq 0\}.$$

We explicitly denote the homeomorphism f only when it is necessary. We define the local unstable set of f at $x \in X$ for $\delta \geq 0$ by

AMS–MOS classification numbers: Primary 54H20. Secondary 58F15
Received by the editors on November 3, 1985.