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THE ROGERS–RAMANUJAN IDENTITIES WITHOUT JACOBI'S TRIPLE PRODUCT

GEORGE E. ANDREWS

ABSTRACT. We provide polynomial identities which converge to the Rogers–Ramanujan identities. These identities naturally involve the partial products for the related infinite products. Hence Jacobi's triple product identity is never required.

1. Introduction. For many years it was an open question whether a bijective proof could be given for the Rogers–Ramanujan identities. In 1980, A. Garsia and S. Milne [**6**], [**7**] gave the first bijective proof using what has since become called the Garsia–Milne Involution Principle. Subsequently D. Bressoud and D. Zeilberger [**5**] gave an alternative bijective proof; however it also relied on the Garsia– Milne Involution Principle. Indeed, given the known analytic proofs of the Rogers–Ramanujan identities it seems that the Involution Principle is inherently involved; this is because all the known proofs actually establish

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q;q)_\infty} \sum_{\lambda=-\infty}^{\infty} (-1)^\lambda q^{\lambda(5\lambda+1)/2} \tag{1.1}$$

and

$$\sum_{n=0}^{\infty} \frac{q^{n^2}+n}{(q;q)_n} = \frac{1}{(q;q)_\infty} \sum_{\lambda=-\infty}^{\infty} (-1)^\lambda q^{\lambda(5\lambda+3)/2}, \tag{1.2}$$

where

$$(A;q)_n = \prod_{m=0}^{\infty} (1-Aq^m)/(1-Aq^{m+n}) \tag{1.3}$$

$(= (1-A)(1-Aq)\cdots(1-Aq^{n-1})$, when n is a nonnegative integer),

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