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COMPLETENESS PROPERTIES OF HYPERSPACES OF COMPACT FUZZY SETS

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0. Introduction. On an arbitrary uniform space there are two types of "compactlike" fuzzy sets which are widely used in applications: u.s.c. fuzzy sets with compact support (we denote this collection $\Phi_c(X)$) and u.s.c. fuzzy sets with compact levelsets (we denote this collection $\Phi_W(X)$) [**2**], [**12**]. Always $\Phi_c(X) \subset \Phi_W(X)$ but the converse holds only if X itself is compact.

In the first part of our paper we prove that for the global fuzzy hyperspace structure [**8**], [**9**] the completeness of X is equivalent to the completeness of $\Phi_c(X)$ and to either the completeness or the ultracompleteness of $\Phi_W(X)$ [**6**], [**7**].

In the second part we then prove the rather surprising result that the completion of $\Phi_c(X)$ [**7**] is isomorphic to $\Phi_W(\hat{X})$ where $\hat{X}$ denotes the completion of X.

These results not only generalize K. Morita's results on hyperspace of compact subsets [**11**] to the setting of fuzzy hyperspaces of "compactlike" fuzzy subsets but moreover via the isomorphism of the uniform modification of $\Phi_c(X)$ and $\Phi_W(X)$ with hyperspaces of closed subsets of $X \times [0,1]$ [**9**], they also include an extension of K. Morita's classical results to classes of closed subsets of $X \times [0,1]$ which are in general not compact.

1. Preliminaries. In this section we shall recall notations and basic concepts which are used throughout the rest of the paper.

I denotes the unit interval, I_0 stands for $]0,1]$ and I_1 stands for $[0,1[$.
The characteristic function of a subset $Y \subset X$ is denoted 1_Y.

If X is a topological space then contrary to usual notation in hyperspace theory we shall put 2^X for all subsets of X and $\mathcal{F}(X)$ for all closed subsets of X [**9**].

For notations and basic results on prefilters and convergence we refer

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