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LIFTING OF ROTUNDITY PROPERTIES FROM E TO $L^p(\mu, E)$

GIOVANNI EMMANUELE AND ALFONSO VILLANI

ABSTRACT. We consider some rotundity properties which are extensions of the uniform rotundity and show that these properties lift from the Banach space E (or from the conjugate Banach space E^*) to the Lebesgue–Bochner function space $L^p(\mu, E)$ (or to $(L^p(\mu, E))^*$), $1 < p < \infty$. We make no assumption on E^*; in particular, we do not assume that E^* has the Radon–Nikodym property.

0. Introduction. In their paper [**16**] Smith and Turett give several interesting results about the geometry of the Lebesgue–Bochner function spaces $L^p(\mu, E)$. In particular they show that the following statement holds.

THEOREM 0. *Let $(S, \sum, \mu)$, a finite measure space, and E, a Banach space, be given. Assume that E^*, the conjugate space of E, satisfies the Radon–Nikodym property. Then $L^p(\mu, E)$, $1 < p < \infty$, is weakly uniformly rotund if and only if E is.*

One of the purposes of this paper is to prove the above result without any assumption on E^*. (By the way, it is unknown up to now whether the weak uniform rotundity of a Banach space E implies that E^* has the Radon–Nikodym property). Moreover, we consider three other geometric properties, namely *weak local uniform rotundity weak* uniform rotundity* (in a conjugate space) and *weak* local uniform rotundity,* and we show that they lift from E (or E^*) to $L^p(\mu, E)$ (or $(L^p(\mu, E))^*$). Also for these properties we will make no assumption on E^*. It is worth noting that assuming the Radon–Nikodym property for E^* would be an effective restriction in this case (see Remark 4 at the end of the paper).

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