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ON THE SPACE ℓ/c_o

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ABSTRACT. In this paper we correct a mistake contained in [3] and we improve and give simpler proofs of some of the results contained there. We also give a very simple proof of the fact (included in Theorem 5.6 of [5]) that the dual of every complemented subspace of ℓ^∞/c_o is isomorphic to $(\ell^\infty)'$.

Introduction and notation. If T is a completely regular topological space, βT is its Stone-Cech compactification; if S is a locally compact topological space, αS is its one-point compactification. We recall some facts about ℓ^∞ and ℓ^∞/c_o.

ℓ^∞ is isometric to $C(\beta N)$ and ℓ^∞/c_o is isometric to $C(\beta N\backslash N)$ (cf. [3]).

ℓ^∞ is a P_1-space, that is, it is complemented in every Banach space which contains it with a norm-one projection; $(\ell^\infty)' = \ell^1 \oplus c_o^\perp$ (cf. [2]).

We use $=$ for "isomorphic to" and $\equiv$ for "isometric to".

If $E_n, n \in N$, are Banach spaces, then

$$(\oplus_n E_n)_p = \{(x_n) | x_n \in E_n \text{ and } ||(x_n)||_p = (\sum_n ||x_n||^p)^{1/p} < \infty\}, \quad 1 \le p \le \infty,$$

$$(\oplus_n E_n)_\infty = \{(x_n)|x_n \in E_n \text{ and } ||(x_n)||_\infty = \sup_n ||x_n|| < \infty\}$$

and $(\oplus_n E_n)_{c_o}$ is the closed subspace of $(\oplus_n E_n)_\infty$ formed by the sequences (x_n) such that $\lim_n ||x_n|| = 0$.

It is easy to show that $(\oplus_n E_n)'_p = (\oplus_n E'_n)_{p'}$ if $1 \le p < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$ and $(\oplus_n E_n)'_{c_o} = (\oplus_n E'_n)_1$, but it is false that $(\oplus_n E_n)'_\infty = (\oplus_n E'_n)_1$ in general (for example, consider the case when the E'_ns are Banach spaces with separable dual).

If Γ is a set of indices let $c_o(\Gamma) = \{(x_\alpha)_{\alpha\in\Gamma} | x_\alpha \in C$ and for any $\varepsilon > 0 |x_\alpha| > \varepsilon$ only for a finite number of indices $\}$.

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