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FOURIER TRANSFORM FOR INTEGRABLE BOEHMIANS

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ABSTRACT. Basic properties of the Fourier transform for intgrable Boehmians are discussed. An inversion theorem is proved.

Introduction. The Fourier transform for Boehmians has been defined independently by J. Burzyk (oral communication) and D. Nemzer [**5**]. The definition given by J. Burzyk is very general and in this case the Fourier transform of a Boehmian is not necessarily a function (like the Fourier transform of a tempered distribution). D. Nemzer was particularly interested in the Fourier transform of Boehmians with compact support. This note will discuss basic properties of the so-called integrable Boehmians. In this case the Fourier transform is always a continuous function and has all basic properties of the Fourier transform in $\mathcal{L}_1$. In particular, we will prove an inversion theorem which has the form of a classical theorem in $\mathcal{L}_1$.

1. Integrable Boehmians. A general construction of Boehmians was given in [**2**]. In this note we are interested in a special case of that construction. Denote by $\mathcal{L}_1$ the space of complex valued Lebesgue integrable functions on the real line $\mathbf{R}$. By $||\cdot||$ we mean the norm in $\mathcal{L}_1(||f|| = \int_R |f(x)|dx)$. If $f, g \in \mathcal{L}_1$ then the convolution product $f * g$, i.e.,

$$(f * g)(x) = \int_R f(u)g(x-u)du,$$

is an element of $\mathcal{L}_1$ and $||f * g|| \leq ||f|| \cdot ||g||$.

A sequence of continuous real functions $\delta_n \in \mathcal{L}_1$ will be called a *delta sequence* if

$$\begin{cases} \int_R \delta_n(x)dx = 1 & \text{for every } n \in N, \\ ||\delta_n|| < M & \text{for some } M \in \mathbf{R} \text{ and all } n \in N, \\ \lim_{n\to\infty} \int_{|x|>\varepsilon} |\delta_n(x)|dx = 0 & \text{for each } \varepsilon > 0. \end{cases}$$

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