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PERIODIC SOLUTIONS OF DIFFERENTIAL-DELAY EQUATIONS WITH MORE THAN ONE DELAY

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Introduction. In this paper we prove the existence of nontrivial periodic solutions of certain differential-delay equations with more than one delay. The method of proof involves techniques which have been used to study differential-delay equations with a single delay, and part of our motivation is to show how these techniques can be generalized. Our results also imply a nonuniqueness result for periodic solutions of some differential-delay equations with more than one delay which have been studied by R.D. Nussbaum.

In [**5**] Nussbaum studies the differential-delay equation

$$x'(t) = -\alpha f(x(t-1)), \tag{0.1}$$

where α is a positive parameter and f is an odd function ($f(-x) = -f(x), \forall x$) which decays like x^{-r} at infinity and satisfies $xf(x) \geq 0$ for all x. Nussbaum's original motivation for studying (0.1) was the case $f(x) = x(1 + |x|^{r+1})^{-1}$ for which (0.1) has been suggested as a model for a somewhat more complicated equation which was introduced in a study of physiological control systems [**2,3**]. By now there is a good deal of evidence to suggest that for such f the dynamics of (0.1) are quite complex [**7,8**]. Nussbaum proved (with some additional hypotheses on f, which, nonetheless, included the case $f(x) = x(1 + |x|^{r+1})^{-1}$) that for α large enough (0.1) has a periodic solution the minimal period of which tends to infinity as α tends to infinity. These periodic solutions also have special symmetry properties. The proof involves a careful asymptotic analysis of some of the solutions of (0.1), and while the analysis depends on certain special features of the function f, it appears that the techniques involved can be applied to a much larger class of functions. In fact, this author has been able to use these general methods to study (0.1) for the case in which f decays exponentially at

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