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INTEGRAL REPRESENTATIONS OF LINEAR FUNCTIONALS ON FUNCTION MODULES

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ABSTRACT. An integral representation for linear functionals on function modules is given under the condition that the function module is 'uniformly separable'. This result is a generalization of Riesz' Representation Theorem for linear functionals on $C(X)$. The results apply to spaces of (weighted) vector valued functions and to Grothendieck's G-spaces.

1. Introduction. Function modules were first introduced by R. Godement [**5**], I. Kaplansky [**7**], and M.A. Naimark [**13**] under the name of Continuous Sums. They considered spaces E of functions σ defined on a topological space X with values in given Banach spaces $E_x, x \in X$, satisfying the following axioms:

(1) E is a closed linear subspace of the Banach space $\{\sigma \in \prod_{x\in X} E_x : \sup_{x\in X} ||\sigma(x)|| < \infty\}$, equipped with the norm $||\sigma|| = \sup_{x\in X} ||\sigma(x)||$.

(2) The function $x \mapsto ||\sigma(x)|| : X \to \mathcal{R}$ is upper semicontinuous for every $\sigma \in E$.

(3) $E_x = \{\sigma(x) : \sigma \in E\}$ for every $x \in X$.

(4) E is a $C_b(X)$-module with respect to the multiplication $(f, \sigma) \mapsto f\sigma$ where $(f\sigma)(x) = f(x)\sigma(x)$ and where $C_b(X)$ denotes the algebra of all bounded continuous scalar valued functions on X.

Let us agree to call E a *function module over* X. For a given $x \in X$, the Banach space E_x is called the *stalk over* x.

Function modules are important in the representation theory of C^*-algebras (see Dauns and Hofmann [**3**]). For compact Hausdorff spaces X, the notion of function modules is also equivalent to the notion of spaces of section in a Banach bundle over X (see [**4**] for the details of this equivalence). Examples for function modules are the Banach spaces $C_b(X), C_b(X, F)$ (the space of all continuous functions with values in a given Banach space F) as well as spaces of continuous functions equipped with a weighted norm.

The object of the present note is to study the dual space of a function module. For all the examples mentioned above, integral representations

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