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CONVEX PROGRAMMING WITH SET FUNCTIONS

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ABSTRACT. Concepts of subdifferentials and conjugate functions are extended to convex set functions. Optimality conditions for convex programs with set functions are then characterized by subdifferentials of set functions involved. Duality theorems of Wolfe type for convex programs with set functions are also developed.

1. Introduction. Recently, properties of convexity, differentiability and epigraphs of set functions were investigated in several papers (e.g., [**1**,**2**, and **3**]). These results were then used to characterize the optimal solution of mathematical programming problems with set functions. This paper presents some results following the same direction of those papers but with different approaches.

In this paper, we consider the following convex program with set functions:

$$\min_{\Omega \in \mathcal{S}} F(\Omega) \text{ subject to } G^i(\Omega) \leq 0, \quad i = 1, \ldots, n,$$

where F and each G^i are convex set functions $\mathcal{A} \to \mathbf{R} \bigcup \{\infty\}$, $\mathcal{S}$ is a nonempty convex subfamily of $\mathcal{A}$, and $(X, \mathcal{A}, \mu)$ is a measure space. Concepts of subgradients, subdifferentials, and conjugate functions of convex functions are extended to convex set functions in §2. Subsequently, their basic properties are also explored. In §3, optimality conditions at Ω^* for convex programs with set functions are then characterized by the sum of subdifferentials of all set functions involved at Ω^*, and the normal cone of the set $\mathcal{D} = \mathcal{S} \bigcap (\operatorname{dom} F) \bigcap_{i=1}^n (\operatorname{dom} G^i)$ at Ω^*. This result (see Theorem 3.3) presents general optimality conditions for convex programs with set functions since we include a constrained convex subfamily $\mathcal{S}$ in the setting. Finally, duality theorems of Wolfe type for convex programs with set functions are developed in §4.

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