ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 3, Summer 1987

REMARKS ON GLOBAL BOUNDS OF SOLUTIONS OF PARABOLIC EQUATIONS IN DIVERGENCE FORM

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Introduction. We want to examine certain properties of solutions (understood here as the trajectories in a suitable Banach space) of nonlinear parabolic equations, which could be of use in the study, among other things, of the long time behaviour of these problems. We start with the proof of a variant of the maximum principle (cf. [**8**; Theorems 2 and 5], [**9**; Theorem 2.7] for the linear case) obtained here as a limiting case ($p \to \infty$) of the sub-exponential estimates of solutions in L^p. Next (§2) we prove a theorem concerning global boundedness of the spatial derivatives u_x, first in the one dimensional case ($n = 1$) for equations with bounded perturbation $f(t, x, u, u_x, u_{xx}, u_t)$, then (§3) by different method (and with stronger assumptions) for the general n-dimensional case. The results obtained in this work are linked up by the method of proofs developed under the stimulus of Theorem 3.1 of [**1**], first used in a different context by J. Moser in [**12**].

Preliminaries. Notation. The following standard notation is used:
(a) $\Omega \subset R^n$ is a bounded domain with a suitable smooth boundary;
(b) $R^+ = [0, \infty), D = R^+ \times \Omega, D_T = \{(t, x) : 0 \leq t \leq T, x \in \Omega\}$,
(c) $\langle \cdot, \cdot \rangle$ is the scalar product in R^n,
(d) $|\Omega|$ is the Lebesgue measure of Ω; and
(e) for $x \in R^n$ we write $u_x = (\frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_n})$; and
(f) use the usual notation for the L^p and Sobolev spaces. By convention, all sums are taken from 1 to n, and integrals with unspecified domain are taken over Ω.

The following easy lemma is required several times.

LEMMA 0. *Let* $y \in C^\circ(R^+)$, $x \in C^1(R^+)$, *let* $\alpha \geq 0, \beta, \delta, \lambda > 0$ *and*

Received by the editors on September 23, 1984 and in revised form on September 15, 1985