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ON THE NONNEGATIVITY OF SOLUTIONS OF REACTION DIFFUSION EQUATIONS

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ABSTRACT. Consider the system of reaction diffusion equations

$$\frac{\partial u}{\partial t} = A\Delta u + f(u, \nabla_x u, x, t) \tag{*}$$

where A is a $p \times p$ matrix, $u(x,t)$ is a p-dimensional vector with components $u^{(j)}(x,t), j = 1, \cdots, p$ and where $(x,t) \in R^n \times (0,\infty)$. Motivated by phenomena modeled by (*) in which off-diagonal entries of the matrix A are specifically included, we study the circumstances under which solutions of the initial boundary value problem for (*) in $\Omega \times (0,T)$ (Ω a bounded domain) with either homogeneous Dirichlet or Neumann boundary conditions holding on $\partial\Omega \times (0,T)$, have the property that starting out from nonnegative initial data, they will remain nonnegative for all subsequent times. For the simplest equation modeling multicomponent diffusion [**4**] which corresponds to $f \equiv 0$: $\frac{\partial u}{\partial t} = A\Delta u$ with A a constant positive definite matrix, we show that the property of persistence of nonnegativity for solutions cannot hold unless the off-diagonal entries of A are not present. To obtain a result assuring the persistence of nonnegativity with the off-diagonal entries a_{jk} of A present, we assume that these entries depend on u and $\nabla_x u$ as follows:

$$a_{jk} = u^{(j)}\alpha_{jk}(u, \nabla_x u, x, t) \quad (j \neq k)$$

while the diagonal entries are assumed to be positive and f is suitably structured. This result is applicable to the equations used by Keller and Segel [**10**] to model slime mold aggregation; as well as to more sophisticated models for multicomponent diffusion accompanied by a reaction such as appear in the most general formulation of combustion theory.

1. Introduction. Equations of the form

$$\frac{\partial u}{\partial t} = A\Delta u + f(u, \nabla_x u, x, t), \tag{1.1}$$

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