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IMPLICATIONS FOR SEMIGROUPS EMBEDDABLE IN ORTHOCRYPTOGROUPS

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Malcev [**13**] was the first to give necessary and sufficient conditions for the embeddability of a semigroup in a group. His conditions are countably infinite in number. No finite number of these conditions is sufficient to ensure embeddability. A similar set of conditions, but involving geometrical concepts, was given by Lambek [**12**]. An account of these results, and of the necessary and sufficient conditions for the embeddability of a semigroup in a group due to Pták [**17**], is presented in Chapter 12 of Clifford and Preston [**5**]. The account of Clifford and Preston employs the concept of a free group on a semigroup. If S is any semigroup then the pair (G, γ), where G is a group and $\gamma : S \to G$ is a homomorphism for which $S\gamma$ is a set of group generators for G, is said to be a free group on the semigroup S if it satisfies the following universal property: if H is a group and $\eta : S \to H$ is a homomorphism such that $S\eta$ is a set of group generators for H, then there exists a homomorphism $\theta : G \to H$ such that $\gamma\theta = \eta$. The existence and uniqueness of the free group on a semigroup are established, and it is shown that a semigroup S is embeddable in a group if and only if the canonical homomorphism γ of S into the free group on S is an embedding. Various aspects of the work of Malcev and Lambek have been considered by Bush [**4**], Bouleau [**3**], and Osondu [**15**]. Simpler conditions which are sufficient to ensure the embeddability of a semigroup in a group have been given by Doss [**6**], Trotter [**23**], and Bouleau [**2**]. Schein [**18, 19**] has given necessary and sufficient conditions for a semigroup to be embeddable in an inverse semigroup.

A semigroup which is a union of groups is said to be *completely regular*. A completely regular semigroup S comes naturally equipped with a unary operation of inverse by letting a^{-1}, for $a \in S$, be the inverse of a in the maximal subgroup of S which contains a. The class of completely regular semigroups forms a variety of type $\langle 2, 1\rangle$ which has the defining identities $a(bc) = (ab)c, aa^{-1}a = a, aa^{-1} =$

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