ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 3, Summer 1987

ON TORSION IN GROUPS WHOSE AUTOMORPHISM GROUPS HAVE FINITE RANK

SILVANA FRANCIOSI, FRANCESCO DE GIOVANNI
AND DEREK J.S. ROBINSON

1. Introduction. Our object is to study the effect on the elements of finite order in a group of imposing finiteness conditions on the automorphism group. That some effect is to be expected is suggested by results already in the literature. Almost thirty years ago Baer [1] showed that a torsion group whose automorphism group is finite is itself finite. This result was sharpened by Nagrebeckiĭ [9] who proved that if the automorphism group Aut G of a group G is finite, then the elements of finite order form a finite subgroup of G. Subsequently it was observed that certain apparently weaker finiteness properties are in fact equivalent to the finiteness of Aut G. Thus Robinson [14] showed that if Aut G is a Černikov group, then it is finite. Recently Zimmerman [17] has proved that Aut G will also be finite if it is a countable torsion FC-group with no elements of order 2 or 3.

Here we shall consider properties of the automorphism group which are usually weaker than finiteness but which are strong enough to force Sylow subgroups of the group to be small. The most general property that we consider is that of having finite abelian subgroup rank; this property requires that every abelian subgroup A have finite torsion-free rank $r_0(A)$ and finite p-rank $r_p(A)$ for all primes p. Somewhat stronger is the requirement of finite abelian total rank; for this the total rank

$$r_0(A) + \sum_p r_p(A)$$

of each abelian subgroup A must be finite. Stronger still are the maximal and minimal conditions on abelian subgroups, $\max - ab$ and $\min - ab$.

Of course Nagrebeckiĭ's Theorem is really about abelian-by-finite groups. We shall prove our results for soluble-by-finite groups.

Received by the editors on December 26, 1984 and in revised form on September 26, 1985.
This research was carried out while the first two authors were visitors at the University of Illinois in Urbana and held grants from the C.N.R. (Italy).