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A NOTE ON βD–D

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ABSTRACT. Let $U(\kappa)$ be the space of ultrafilters on the infinite regular cardinal κ and let SL_κ be the generalization of "Solovay's Lemma" SL_ω to κ. Our main result is to show that, assuming SL_ω, every cellular family of fewer than 2^κ open subsets of $U(\kappa)$ has a C^*–embedded selection. Further results are provided which are intended to exhibit the need for assuming SL_κ in the above result.

1. Introduction. In this paper we present some results concerning the C^*–embedding of subsets of βD–D. βD–D (also denoted D^*) denotes the remainder of the Cech–Stone compactification of the discrete space D, and a subset is said to be C^*–embedded if every bounded continuous real–valued function on the subspace can be extended to one on the entire space. For backgound on C^*–embedding and Cech–Stone compactifications the reader is referred to [**5**,**9**].

There are many known related results. Before stating some of the known results, we introduce some notation.

DEFINITION.

(i) A cellular family of subsets of a space is a family, any two members of which are pairwise disjoint.

(ii) A clopen subset is one that is both closed and open.

(iii) If κ is an infinite cardinal, then a uniform ultrafilter on a set of size κ is an ultrafilter all of whose members has cardinality κ. Given a discrete set D of size κ, the subspace of βD–D consisting of the uniform ultrafilters is denoted $U(\kappa)$. Thus, $U(\omega) = \beta N$–N.

(iv) If κ is an infinite cardinal, then Solovay's Lemma for cardinal κ(denoted SL_κ) is the following statement: Suppose λ is an infinite cardinal for which $\lambda < 2^\kappa$. Let $\{F_i\}_{i<\lambda}$ and $\{G_j\}_{j<\lambda}$ be collections of subsets of κ such that if $j < \kappa$ and S is a subset of κ for which $|S| < \kappa$, then $|G_j - \cup_{i\in S} F_i| = \kappa$. Then there is a set $B \subset \kappa$ such that, for any $i < \kappa, |B \cap F_i| < \kappa$ and $|B \cap G_i| = \kappa$.

Spaces of the form $U(\kappa)$ have been studied in detail in [**3**]. Solovay's

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