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AN α–APPROXIMATION THEOREM FOR R^∞–MANIFOLDS

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0. Introduction and preliminaries. Generalizing the CE-approximation theorem of Armentrout [**1,2**] and Siebenmann [**20**] for finite-dimensional manifolds, Ferry proved an α–approximation theorem for Q–manifolds in [**8**] and an α–approximation theorem for manifolds of dimensions ≥ 5 in a joint work with Chapman [**6**].

Recently, the author proved in [**16**] an α–approximation theorem for Q^∞–manifolds: "Given an open cover α of a Q^∞–manifold N, then there is an open cover β of N such that every β–equivalence from a Q^∞–manifold M to N is α–close to a homeomorphism".

It will be shown in this note that such an α–approximation theorem also holds true for R^∞–manifolds. So, the question (NLC 8) in [**9**] has an affirmative answer.

As in [**16**], in the process of proving the main theorem, some results similar to a few properties of Z–sets in Q and ℓ_2–manifold theory will be proved. These include:

(1) relative R^∞–deficient embedding approximation theorem (Theorem 2.3);

(2) unknotting theorem for R^∞–deficient embeddings (Theorem 3.3);

(3) collar theorem (Theorem 4.2); and

(4) R^∞–deficient subsets being strongly negligible (Theorem 5.3).

For standard concepts such as the cone(X) of a topological space X, the mapping cylinder $M(f)$ of a map f, the infinite mapping cylinder $M(f_1, f_2, ...)$ of a sequence of maps $f_i : X_{i-1} \to X_i$, the limitation of a homotopy $H : X \times I \to Y$ by an open cover α of Y, the n^{th}–star $\text{St}^n(\alpha)$ of an open cover α, etc., we refer to [**8**] or [**16**] for more details. All topological spaces are separable.

Throughout this note, let R^∞ be the direct–limit space $\lim_{\to}\{R^n\}$

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