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INFINITE SUMS OF PRODUCTS OF CONTINUOUS q-ULTRASPHERICAL FUNCTIONS

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ABSTRACT. Let $C_n(x;\beta|q)$ be the continuous q-ultra-spherical polynomial and $D_n(x;\beta|q)$ be the q-ultraspherical function of the second kind. By exploiting a special case of the recently found q-Feldheim bilinear sum, the following infinite sums are computed:

$$\sum_{n=0}^{\infty}\left(\frac{(q;q)_n}{(\beta^2;q)_n}\right)^2\frac{1-\beta q^n}{1-\beta}\beta^{n/2}C_n(x;\beta|q)C_n(y;\beta|q)C_n(z;\beta|q),$$

$0<\beta<1,\ 0<q<1$, and

$$\sum_{n=0}^{\infty}\frac{(q;q)_n}{(\beta^2;q)_n}\frac{1-\beta q^n}{1-\beta}(q/\beta)^{n/2}C_n(x;\beta|q)C_n(y;\beta|q)D_n(z;\beta|q)$$

$0<q<\beta<1$.

1. Introduction. Recently, Rahman [8] showed that

$$\begin{aligned}C_n(\cos\theta;\beta|q) = \frac{(1+\beta q^n)(\beta^2;q)_n}{(1+\beta)(q;q)_n}\Big(&Q_n(e^{i\theta};\beta^{\frac12},(\beta q)^{\frac12},\\ &-\beta^{\frac12},-(\beta q)^{\frac12})+Q_n(e^{-i\theta};\beta^{\frac12},(\beta q)^{\frac12},-\beta^{\frac12},-(\beta q)^{\frac12})\Big),\\ &0\le\theta\le\pi\end{aligned}\tag{1.1}$$

and

$$\begin{aligned}D_n(\cos\theta;\beta|q) = i\frac{(1+\beta q^n)(\beta^2;q)_n}{(1+\beta)(q;q)_n}\Big(&Q_n(e^{i\theta};\beta^{\frac12},(\beta q)^{\frac12},-\beta^{\frac12},-(\beta q)^{\frac12})\\ &-Q_n(e^{-i\theta};\beta^{\frac12},(\beta q)^{\frac12},-\beta^{\frac12},-(\beta q)^{\frac12})\Big),0<\theta<\pi,\end{aligned}\tag{1.2}$$

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