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# REFINED TAUBERIAN GAP THEOREMS FOR POWER SERIES METHODS OF SUMMABILITY

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**1. Introduction.** Besides the usual Tauberian gap theorems there are refined forms which, e.g., deal with mixed conditions or lead to summability instead of convergence. Here we are concerned with an instance of the latter kind. Let us assume that $\sum_0^\infty a_n$ is a gap series:

$$a_n = 0, \text{ for } n \neq k_0, k_1, \cdots \tag{1.1}$$

($\{k_n\}$ a given sequence of integers, $0 \leq k_0 < k_1 < \cdots$). Hardy-Littlewood's classical high indices theorem for $A_0$ (Abel's method of summability) states that

$$A_0 - \sum_0^\infty a_n = s \text{ implies } \sum_0^\infty a_n = s$$

if $k_{n+1} \geq ck_n$ for a constant $c > 1$. Now let $p$ be a non-negative integer. Then, according to a special case of a theorem of Korenblyum (see [**11**]),

$$A_0 - \sum_0^\infty a_n = s \text{ implies } C_p - \sum_0^\infty a_n = s \tag{1.2}$$

if

$$k_{n+p+1} \geq ck_n \text{ for a constant } c > 1 \tag{1.3}$$

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