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# STRENGTHENED MAXIMAL FUNCTION AND POINTWISE CONVERGENCE IN $\mathbf{R}^n$, II

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**1. Introduction.** In the first article [**1**] of this title, we introduced a new method for dealing with problems of pointwise convergence in $\mathbf{R}^n$. The simplest problem of this sort is the differentiation problem for integrals, which may be phrased as follows: Find conditions on measurable functions $f$ and sequences of sets $\{E_k(x)\}$ which guarantee that the corresponding sequence of averages of $f$ over each $E_k(x)$ has $f(x)$ for its limit, either at particular values of $x$ or at almost every $x$. The literature on this problem is extensive and dates from Lebesgue; Guzman [**4**] gives a comprehensive survey of developments through the early seventies. Our method in [**1**] led to a precise relationship between a type of regularity condition on the sequences of sets and integrability properties of the function needed for these problems in $\mathbf{R}^n$. Here we pursue a similar program in an abstract setting. This approach not only allows us to apply our methods in other situations but also gives a more precise analysis of the special case we treated in [**1**]. It also exposes some interesting points which were previously concealed.

For the problem of almost everywhere convergence, the difficulties we encounter are purely technical. Our earlier work was based on the Hardy–Littlewood maximal function; its precise continuity properties are reflected in norm inequalities involving well–known spaces. Consequently, it seemed natural to give norm inequalities for our strengthened maximal function in [**1**]. Such estimates here would require more hypotheses than we care to assume and would involve unduly complicated norms. Instead, we construct a kernel on the multiplicative group $\mathrm{R}_+$ and show that the averaged decreasing rearrangement of our strengthened maximal function is bounded by the convolutions of this kernel with the decreasing rearrangement of the original function. In specific applications, the appropriate norm inequalities in rearrange-

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