ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 3, Summer 1987

MODULI OF CONTINUITY AND GENERALIZED BCH SETS

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1. Introduction. Let $\Lambda = \{|z| < 1\}$ and $C = \{|z| = 1\}$. For $\omega = \omega_h$, the modulus of continuity of a bounded complex–valued function h on $[0, 2\pi]$, the class of ω–sets is defined to be the subclass of closed sets K of linear measure 0 in C satisfying

$$\sum_k \omega(|I_k|) < \infty, \tag{1.1}$$

where (I_k) is an enumeration of the component arcs of $C\backslash K$ and $|I_k|$ is the length of I_k. When ω is *equivalent* to the modulus of continuity $\nu(t) = t\log(2\pi e/t)$ (that is, $m\omega \leq \nu \leq M\omega$ for some $m, M > 0$), then (1.1) is referred to as the Carleson condition and the ω–sets are called the BCH (Beurling–Carleson–Hayman) sets.

The BCH sets first arose in the characterization by Beurling [**4**] and Carleson [**5**] of the boundary zero sets of the functions in the class Λ_ω when $\omega(t) = t^\alpha, 0 < \alpha \leq 1$. By definition, Λ_ω is the class of continuous functions f on $\overline{\Lambda} = \{|z| \leq 1\}$ that are analytic in Λ and satisfy

$$|f(z) - f(w)| \leq c\omega(|z - w|), \quad z, w \in \Lambda,$$

for some $c > 0$. Recently, Shirokov [**15**] generalized the result of Beurling and Carleson by characterizing the complete zero sets $Z(f)$ of functions f in Λ_ω for arbitrary moduli of continuity ω.

* The second and third authors were supported in part by the National Science Foundation.
** Received by the editors on November 16, 1984 and in revised form on August 26, 1985.