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REPELLERS IN REACTION-DIFFUSION SYSTEMS

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ABSTRACT. A technique for discovering when an invariant set for a reaction-diffusion system is a repeller in a certain strong sense is studied. The criterion is based on a weakening of the standard requirements for Liapunov functionals for repellers. The analysis is motivated by the coexistence question for several interacting species in mathematical biology.

1. Introduction. In biological applications, see [**4**], the following initial/boundary value problem for a system of reaction-diffusion equations in $D \times \mathbf{R}_+$ is often encountered:

$$\partial u_i/\partial t = \mu_i \Delta u_i + u_i f_i(u), \tag{1.1a}$$

$$\partial u_i/\partial \nu = 0 \quad (\text{on } \partial D \times \mathbf{R}_+), \tag{1.1b}$$

$$u(x,0) = u_0(x) \quad (x \in \overline{D}), \tag{1.1c}$$

where $1 \le i \le n$ and $u = (u_1, \ldots, u_n)$. Here, D is a bounded domain in $\mathbf{R}^m$ with smooth boundary, $\partial/\partial\nu$ denotes differentiation along the normal to ∂D and Δ is the Laplacian. The function u_i is the density of the i^{th} population, and the boundary condition (1.1b) requires that there should be no migration across ∂D. Only non-negative solutions are of interest, and it should be noted that from the form of the equations, each of the sets $u_i(x) = 0 (x \in \overline{D})$ is forward invariant.

The problem considered here, that of obtaining criteria for the long term survival of the species, is one of the most fundamental from the point of view of applications. However, precisely how 'survival' ought to be interpreted is not clear, and indeed for the much simpler model based on the corresponding ordinary differential equations, there have been a number of definitions proposed in the literature. The definition that will be used here is as follows: the system (1.1) will be said to be *permanently coexistent* if and only if there exists an $\varepsilon > 0$ such that,

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