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# ON SUMS OF SIXTEEN SQUARES

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ABSTRACT. The author shows that the function $r_{16}$, which counts the totality of representations of a natural number by sums of sixteen squares, is expressible entirely in terms of real divisor–functions.

**1. The main result.** It is the purpose of this paper to prove the following formula for the number $r_{16}(n)$ of ways of representing a positive integer $n$ by sums of sixteen squares:

(1)

$$\begin{aligned} r_{16}(n) = \frac{32}{17}\big[&\sigma_7(n) - 2\sigma_7(n/2) + 2^8\sigma_7(n/4) \\ &+ (-1)^{n-1}16\big(2^{3b(n)}\sigma_3(0(n)) \\ &+ 16\sum_{d=1}^{n-1}(-1)^d d^3 \sum_{k=1} 2^{3b(n-kd)}\sigma_3(0(n-kd))\big)\big], \end{aligned}$$

where for positive integers $r, m, \sigma_r(m)$ denotes the sum of the $r$th powers of all positive divisors of $m$, otherwise $\sigma_r(x) := 0; b(n)$ denotes the exponent of the highest power of 2 dividing $n$; and, $0(n)$ is then defined by the equation $n = 2^{b(n)}0(n)$. (By convention the sum on the right side of (1), indexed by $k$, extends over all positive integral values of $k$ for which $n - kd > 0$.)

Proof of (1): We, first of all, recall that the modular function $f$ is defined on the open unit disk of the complex plane (i.e. $x \in C\ |x| < 1$) by:

$$f(x) = x^{1/24}\prod_{1}^{\infty}(1 - x^n).$$

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