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ABSTRACT. Amalgams are spaces of functions on locally compact groups; the condition for membership of a function in an amalgam is a mixture of local and global conditions on the function. This paper concerns the extent to which the classical theory of thin sets in discrete abelian groups transfers to the context of amalgams on noncompact, locally compact, abelian groups. The classes of thin sets considered here are the $\Delta(p)$ sets and the p–Sidon sets. The transference is quite satisfactory for $\Delta(p)$ sets, while the results for p–Sidon sets are less complete. In both cases, the transference yields conclusions about pseudodilation of thin sets in discrete groups.

1. Introduction. Denote the unit circle by T and the real line by R. It is well known that if a function in $L^1(T)$ has its Fourier coefficients supported by a sufficiently thin set, then the function also belongs to the smaller spaces $L^p(T)$ for all indices p in the intervals $(1, \infty)$. The analogous statement also holds for functions in $L^1(R)$ with thinly supported Fourier transforms [**7**, Theorem 3], but this conclusion is less satisfying, because the spaces $L^p(R)$, for $1 < p < \infty$, are merely different from $L^1(R)$ rather than being smaller than the latter space. The point of the present paper is that replacing the L^p-spaces by amalgams based on them leads to more satisfactory results for certain types of thin sets in R and other nondiscrete, noncompact, locally compact, abelian groups. These results include new conclusions about lacunarity and L^p-spaces on such groups. Moreover, they also lead to new examples of thin sets in discrete abelian groups.

We will discuss the case of the group R in this section, and deal with the general situation in §2 and §3. Denote the integer lattice in R by Z. Let I be the half-open interval $[0, 1)$, and for each integer n, let

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