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SUPERNORMAL CONES AND FIXED POINT THEORY

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1. Recently, we defined in [**5**] an interesting class of convex cones imposed by the theory of Pareto optimization, by the study of critical points of dynamical systems and by the study of conical support points. This convex cone was called, in [**5**], "nuclear cone" since every normal cone in a nuclear space is a nuclear cone in our sense. We also note that in [**8**], this cone was called "supernormal" cone, since we observed that the nuclear cone, by its properties, seems to be a reinforcement of concept of normal cone. We adopt in this note the concept of supernormal cone. Applications of supernormal cones may be found in [**5, 6, 7, 8**].

In this paper, we give an interesting application of supernormal cones to fixed point theory.

2. We will use the concept of locally convex space defined by Treves [**14**], that is, a couple $(E, \mathrm{Spec}(E))$, where E is a real vector space and $\mathrm{Spec}(E)$ is a set of seminorms on E such that

(1°) $\forall \lambda \in \mathbf{R}_+$ and $\forall p \in \mathrm{Spec}(E)$, $\lambda p \in \mathrm{Spec}(E)$;

(2°) If $p \in \mathrm{Spec}(E)$ and q is a seminorm on E such that $q \le p$, then $q \in \mathrm{Spec}(E)$; and

(3°) $\forall p_1, p_2 \in \mathrm{Spec}(E)$, $\sup(p_1, p_2) \in \mathrm{Spec}(E)$, where $\sup(p_1,p_2)(x) = \sup(p_1(x), p_2(x))$, for any $x \in E$.

If $\mathrm{Spec}(E)$ is given, then there exists a locally convex topology τ on E such that $E(\tau)$ is a locally convex vector space and a seminorm p on E is τ-continuous if and only if $p \in \mathrm{Spec}(E)$.

A subset $\mathcal{B} \subset \mathrm{Spec}(E)$ is called a basis of $\mathrm{Spec}(E)$ if and only if, for every $p \in \mathrm{Spec}(E)$, there exists $q \in \mathcal{B}$ and a real number $\lambda > 0$ such that $p \le \lambda q$. We suppose that the $\mathrm{Spec}(E)$ has a Hausdorff basis, that is, $\ker \mathcal{B} = \{0\}$, where

$$\ker \mathcal{B} = \{ x \in E | p(x) = 0, \forall p \in \mathcal{B} \}.$$

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