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THE ZARISKI TOPOLOGY FOR DISTRIBUTIVE LATTICES

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ABSTRACT. Our purpose in this paper is to study an intrinsic topology for distributive lattices which by its very definition is analogous to the classical Zariski topology on rings. As in the case of rings, the Zariski topology is the coarsest topology making solution sets of polynomials closed. In other words, the Zariski closed sets are generated from a subbase consisting of all sets of the form $\{z \in L : p(z) = c\}$ where $p(x)$ is a polynomial over L and c is an arbitrary but element from L.

Although the name of this topology for lattices is new, the Zariski topology has appeared, usually unnamed and implicitly, in a variety of guises and formulations in lattice theory over the years. Recently, under different formulations, it was used effectively by R. Ball in [**3**] and by H. Bunch in [**7**].

Out context for studying intrinsic topologies on distributive lattices was set by Frink in [**9**] where he stated, "Many mathematical systems are at the same time lattices and topological spaces. It is natural to inquire whether the topology in such systems in definable in terms of the order relation alone." Seeking systems which are at the same time lattices and topological spaces, one must begin with R, the real line with its usual topology and order, along with two of its substuctures I, the closed unit interval of R, and 2, the chain consisting of the numbers 0 and 1, and then go on to form all finite and infinite Cartesian products of such systems using the product order and the Tychonoff topology. Given this collection of mathematical systems, is it possible to find one intrinsic topology–really, topology definition scheme–which will define the topology exclusively from the order?

For $R, I, 2$ and any chain, the interval topology gives the correct

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