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THE CENTRAL LIMIT QUESTION UNDER ρ-MIXING

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ABSTRACT. An earlier construction of a (non-trivial) strictly stationary ρ-mixing random sequence that fails to satisfy the central limit theorem, is refined here in order to try to have the fastest possible "mixing rate" for ρ-mixing, depending on the "moment properties" of the r.v.'s. In particular, the examples here show that when only finite second moments are assumed, for the central limit theorem the mixing rate $\sum\rho(2^n) < \infty$ used by Ibragimov is essentially as slow as permissible.

1. Introduction. First we define some notation. Log denotes the natural logarithm, and $\log^+ x := \max\{0, \log x\}$. The indicator function of a set S is denoted by I_S. The notation $a \ll b$ means $a = O(b)$. The notation $a \sim b$ means $\lim a/b = 1$. The greatest integer $\leqq x$ is denoted by $[x]$. A sequence $(a_n, n = 1, 2, \ldots)$ of positive numbers is said to be "slowly varying" as $n \to \infty$ if $\lim_{n\to\infty}[\sup_{n\leqq m\leqq 2n} a_m]/[\inf_{n\leqq m\leqq 2n} a_m] = 1$. When a subscript itself is of the form a_n, it will be written as $a(n)$. The notation $\mathscr{L}_2(\,\cdot\,)$ refers only to real-valued random variables (instead of general complex-valued ones).

Suppose $X := (X_k, k \in \mathbf{Z})$ is a strictly stationary sequence of real-valued random variables on a probability space $(\Omega, \mathscr{F}, P)$. For $-\infty \leqq J \leqq L \leqq \infty$ let $\mathscr{F}_J^L$ denote the σ-field of events generated by the random variables $(X_k, J \leqq k \leqq L)$. For any two σ-fields $\mathscr{A}$ and $\mathscr{B} \subset \mathscr{F}$, define the "maximal correlation" **[8, 12]** by

$$\rho(\mathscr{A}, \mathscr{B}) := \sup|\mathrm{Corr}(f, g)| \qquad f \in \mathscr{L}_2(\mathscr{A}),\ g \in \mathscr{L}_2(\mathscr{B}).$$

For each $n = 1, 2, 3, \ldots$ define the dependence coefficient $\rho(n) := \rho(\mathscr{F}_{-\infty}^0, \mathscr{F}_n^\infty)$. By our assumption of stationarity, $\rho(n) = \rho(\mathscr{F}_{-\infty}^J, \mathscr{F}_{J+n}^\infty)$ $\forall J \in \mathbf{Z}$. Also, obviously the sequence $\rho(n), n = 1, 2, \ldots$ is non-increasing as n increases. The stationary random sequence $X := (X_k)$ is said to be "ρ-mixing" **[18]** if $\rho(n) \to 0$ as $n \to \infty$.

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