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POINTWISE CONVERGENCE OF CAUCHY SEQUENCES

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Many problems in analysis require the introduction of a linear space of functions equipped with a norm appropriate to the application. It is often desirable in such situations that convergence in norm imply pointwise convergence on subsets of the common domain.

The purpose of this paper is to extend the results of **[6]**, where pointwise convergence of convergent sequences was studied, to Cauchy sequences. Our primary tool is convergence on a filter. Specifically, we characterize filters which guarantee the desired convergence properties.

Let G be a linear space of functions with a common domain S. Let X denote the formal linear span of S. It was shown in **[4]** that every seminormed topology on G is equivalent to the topology of convergence on a filter of subsets of X.

In **[6]**, Brace and Thomison introduced the notions of uniform, subuniform, pointwise and subpointwise convergence and investigated conditions under which convergence on a filter implied any of the four. In this paper, we extend these notions to Cauchy sequences of functions in G. We also investigate conditions under which completions of G preserve these properties.

§I contains the basic definitions and discusses the relationship between various notions of convergence. §II investigates certain conditions which guarantee the various convergence properties. §III characterizes conditions under which completions of G retain the convergence properties. §IV is devoted to examples.

We use the notation of **[5]**, **[6]** and **[9]**. Proofs of all results are at the end of the sections.

I. Several notions of convergence. In the following, $\mathfrak{F}$ is a filter in a set S. The functions are scalar valued and have S as their common domain; G is a linear space composed of such functions. A filter in a linear space X will always be assumed to possess a basis consisting of balanced, convex sets.

DEFINITION I.1. **[6]** A sequence $\{f_n\}$ converges to f_0 uniformly (pointwise) on $\mathfrak{F}$ when there is a set F in $\mathfrak{F}$ such that $\{f_n\}$ converges uniformly

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