ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 1, Winter 1987

FOCAL POINTS OF NONLINEAR EQUATIONS-A DYNAMICAL ANALYSIS

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Introduction. We consider the $2n^{th}$ order scalar nonlinear differential equation.

$$x^{(2n)}(t) = f(t, x(t)) \tag{E}$$

where f is continuous on $[0, \infty) \times R$. Let α be a real number $\geqq 0$, and k a natural number with $1 \leqq k \leqq n - 1$. The $(2k, 2(n - k))$ focal point of α for the equation (E) is the smallest $\beta > \alpha$ for which there exists a non-trivial solution of (E) satisfying the boundary conditions

$$\begin{aligned} x^{(i)}(\alpha) &= 0, \qquad 0 \leqq i \leqq 2k - 1 \text{ and} \\ x^{(i)}(\beta) &= 0, \qquad 2k \leqq i \leqq 2n - 1. \end{aligned} \tag{1}$$

For linear equations there is a large bibliography concerning focal points and conjugate points. More relevant to the present work, there are several studies on the relation between the non-existence of solutions to two point boundary value problems (disfocality, disconjugacy) and the existence of monotone solutions having prescribed asymptotic behaviour. We refer to Elias **[2]** in the linear case, and Edelson, Kreith **[4]** in the nonlinear case. Unlike the linear problem, the existence of monotone solutions in the nonlinear case has frequently been established by means of topological methods such as fixed point theorems, which may give less information but are more generally applicable.

Motivated by these considerations we will study the properties of focal point trajectories of nonlinear equations. Specifically, we will show that for the class of equations under consideration, in the disfocal case there must exist monotone solutions which we will call focal asymptotic.

DEFINITION. A trajectory $x(t)$ is said to be $(2k, 2(n - k))$ focal asymptotic on $[\alpha, \infty)$ if it satisfies the conditions

Received by the editors on June 11, 1984, and in revised form on March 12, 1985.