ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 1, Winter 1987

$\mathfrak{F}$-PROJECTORS IN LOCALLY FINITE GROUPS

M. R. DIXON

ABSTRACT. The author discusses the class $\mathscr{X}$ of countable locally finite-solvable groups with min-p for all primes p. It is shown that if $\mathfrak{F}$ is a saturated formation which contains no non co-Hopfian groups then the $\mathfrak{F}$-projectors of a group $G \in \mathscr{X}$ are all conjugate if and only if the group G has only countably many such $\mathfrak{F}$-projectors.

1. Introduction. A group G is said to be locally finite-solvable if every finite subset of elements of G is contained in a finite solvable subgroup of G. In this paper we shall be concerned with the class $\mathfrak{X}$ of all countable locally finite-solvable groups with min-p for all primes p, which was first studied in [1]. Here a group G is said to have min-p if every p-subgroup of G has the minimal condition on subgroups. The structure of groups in the class $\mathfrak{X}$ has been well documented in [1], [4] and [6, chapter 3].

In [4] we obtained a theory of saturated formations in the class $\mathfrak{X}$. For the sake of completeness we now describe this theory. If G is in the class $\mathfrak{X}$ then G will be called an $\mathfrak{X}$-group. Suppose $\mathfrak{Z}$ is a QS-closed subclass of $\mathfrak{X}$; that is every $\mathfrak{Z}$-group in an $\mathfrak{X}$-group, and every section of a $\mathfrak{Z}$-group is a $\mathfrak{Z}$-group. Let π denote a non-empty set of primes and, for each $p \in \pi$, let $f(p)$ be a subclass of $\mathfrak{Z}$ satisfying

(i) $f(p)$ is Q-closed

(ii) If $G \in \mathfrak{Z}$ and

$$N = \bigcap \{C_G(H/K) \mid H/K \text{ is a } p\text{-chief factor of } G \text{ such that } G/C_G(H/K) \in f(p)\}$$

then $G/N \in f(p)$.

The saturated $\mathfrak{Z}$-formation defined locally by f is then the class of groups:

$$\mathfrak{F} = \mathfrak{F}(f) = \mathfrak{Z} \cap \mathfrak{S}_\pi \cap \bigcap_{p \in \pi} \mathfrak{S}_{p'}, \mathfrak{S}_p f(p),$$

where $\mathfrak{S}_\pi$ denotes the class of locally finite-solvable π-groups. Moreover

Received by the editors on December 10, 1984, and in revised form on May 14, 1985.