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# ON THE AZIMI-HAGLER BANACH SPACES

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ABSTRACT. We study the $X_\alpha$ spaces constructed by Azimi and Hagler as examples of hereditarily $l_1$ spaces failing the Schur property. We show that each complemented non weakly sequentially complete subspace of $X_\alpha$ contains a complemented isomorph of $X_\alpha$, and that $X_\alpha$ and $X_\beta$ are isomorphic if and only if they are equal as sets.

Azimi and Hagler [1] have introduced a class of Banach spaces, the $X_\alpha$ spaces. Each of the spaces is hereditarily $\ell_1$ and yet fails the Schur property. In this paper we discuss the isomorphic classification of the $X_\alpha$ spaces and show that each non weakly sequentially complete complemented subspace of an $X_\alpha$ space $X$ contains a complemented isomorph of $X$. This lends credence to the conjecture that the $X_\alpha$ spaces are primary, that is, that if $X_\alpha = Y \oplus Z$, then either $Y$ or $Z$ is itself isomorphic to $X_\alpha$. Indeed, a technique for showing that a space $W$ is primary is to show first that if $W = Y \oplus Z$, then either $Y$ or $Z$ contains a complemented isomorph of $W$ and then to use a decomposition method, based either on $W$ being isomorphic to some infinite direct sum $\Sigma \oplus W$ [5] or on knowledge that either $Y$ or $Z$ is isomorphic to its Cartesian square [3]. In the case of the $X_\alpha$ spaces, Azimi and Hagler [1] showed that $X_\alpha$ is of codimension one in its first Baire class, so that if $X_\alpha = Y \oplus Z$, then precisely one summand is weakly sequentially complete. Thus our result accomplishes the first step in this program. Unfortunately, by the same dimension argument, the summand containing $X_\alpha$ is not isomorphic to its square, and $X_\alpha$ is not isomorphic to any infinite direct sum $\Sigma \oplus X_\alpha$. In the case of James' quasi-reflexive space $J$, Casazza [2] was able to overcome difficulties of this type, and showed $J$ to be primary. Some of our techniques are similar to those used by Casazza in [2]. Our terminology is generally the same as that of [1] or [4], and at several points in the analysis we use perturbation arguments such as Proposition 1.a.9 of [4].

The $X_\alpha$ spaces are defined as follows. Let $\alpha = \{\alpha_i\}_{i=1}^{\infty}$ be a sequence of real numbers satisfying

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