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# GROUPS SATISFYING THE WEAK CHAIN CONDITIONS FOR NORMAL SUBGROUPS

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**1. Introduction.** A group $G$ is said to satisfy the weak maximal condition for normal subgroups if it does not contain an infinite ascending chain $G_1 < G_2 < \cdots < G_i < G_{i+1} < \cdots$ of normal subgroups such that each of the indices $|G_{i+1}: G_i|$ is infinite. The weak minimal condition for normal subgroups is defined similarly. We denote these properties by Max $- \infty$ and Min $- \infty$ for normal subgroups.

Groups satisfying the weak chain conditions for normal subgroups were first considered by L. A. Kurdačenko. In **[6]** he began the study of locally nilpotent groups with these conditions and obtained full information in the periodic case and in the torsion-free case, respectively.

THEOREM (Kurdačenko **[6]**). (i) *A periodic locally nilpotent group satisfies the condition* Max $- \infty$ (*resp.* Min $- \infty$) *for normal subgroups if and only if it is a Černikov group.*

(ii) *A torsion-free locally nilpotent group satisfies the condition* Max $- \infty$ (*resp.* Min $- \infty$) *for normal subgroups if and only if it is a nilpotent minimax group.*

Recently Kurdačenko **[8]** was able to give necessary and sufficient conditions for an arbitrary locally nilpotent group to satisfy the condition Min $- \infty$ for normal subgroups. For a short survey of results in this direction, in particular with respect to Kurdačenko's paper **[7]** on groups satisfying the weak chain conditions for subnormal subgroups, see Curzio **[2]**.

In this note we are concerned with the effect of the weak chain conditions for normal subgroups within the class of (locally) solvable groups. Our first result provides the tool for short proofs of the remaining theorems.

THEOREM A. *If the group $G$ satisfies the condition* Max $- \infty$ (*resp.* Min $- \infty$) *for normal subgroups and $H$ is a subgroup of $G$ with finite index, then $H$ satisfies the condition* Max $- \infty$ (*resp.* Min $- \infty$) *for normal subgroups.*

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