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A SIMPLE CHARACTERIZATION OF THE CONTACT SYSTEM ON $J^k(E)$*

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ABSTRACT. In this note we give an invariant characterization of the contact system of $J^k(E)$ where (E, π, M) is a fibred manifold. This characterization generalizes one given in reference **[1]** for the case where $k = 1$. It affords a simple coordinate free proof that a section σ of $(J^k(E), \pi^k_M, M)$ is the k-jet extension of a section of (E, π, M) if σ annihilates the contact system **[2]**.

1. The First order Case. Let (E, π, M) denote a fibred manifold with total space E, projection π and base space M. The *k-jet* bundle of local sections of (E, π, M), denoted by $J^k(E)$, has a natural fibred manifold structure over $J^\ell(E)$ for $\ell < k$ and over E and M. The canonical projections π^k_ℓ: $J^k(E) \to J^\ell(E)$, π^k_E: $J^k(E) \to E$ and π^k_M: $J^k(E) \to M$ are given by

$$(1)\qquad \begin{aligned} &\text{(a)} && \pi^k_\ell : J^k_x\, s \to J^\ell_x\, s \\ &\text{(b)} && \pi^k_E : j^k_x\, s \to s(x) \end{aligned}$$

and

$$\text{(c)}\quad \pi^k_M = \pi \circ \pi^k_E : j^k_x\, s \to x$$

respectively.

We begin by defining the contact system Ω^1 on $J^1(E)$ as the exterior differential system given pointwise by

$$(2)\qquad \Omega^1|_{j^1_x s} = (\pi^{1*}_E - \pi^{1*}_M s^*)T^*_{s(x)}E.$$

It is easy to verify, from (2), that a section σ of $(J^1(E), \pi^1_M, M)$ defined on $U \subset M$, satisfies $\sigma^*\Omega^1 = 0$ iff $\sigma = j^1 s$ where $s = \pi^1_E \circ \sigma$. To see this, suppose $\sigma = j^1 s$. Then

$$\begin{aligned} \sigma^*\Omega^1|_{j^1_x s} &= j^1 s^*(\pi^{1*}_E - \pi^{1*}_M s^*)T^*_{s(x)}E \\ &= [(\pi^1_E \circ j^1 s)^* - (s \circ \pi^1_M \circ j^1 s)^*]T^*_{s(x)}E \\ &= [s^* - (s \circ id_U)^*]T^*_{s(x)}E = 0. \end{aligned}$$

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