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THE HOLONOMIC IMPERATIVE AND THE HOMOTOPY GROUPOID OF A FOLIATED MANIFOLD

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Since A. Connes in [**1**, **2**] defined the C^*-algebra of a foliation for the purpose of studying index theory on foliated manifolds a number of C^*-algebraists have worked to obtain the necessary geometric prerequisites in order to understand these new examples of C^*-algebras. One of the first conceptual difficulties encountered is the notion of holonomy as defined by C. Ehresmann. Connes based his operator algebra constructions on the holonomy groupoid (or graph) of the foliation as defined by H. E. Winkelnkemper in [**9**]. In fact this groupoid (and its topology) had already been defined in the very general setting of topological foliations by Ehresmann himself [**3**, pp. 130–132]. The C^∞ structure of this groupoid was introduced by J. Pradines in [**6**] although no details or proofs have appeared. Besides the timeliness of his rediscovery of the holonomy groupoid for C^∞ foliations, one of the merits of Winkelnkemper's paper [**9**] is its concrete constructions and concise proofs.

Now, even though holonomy is natural enough from a differential equations point of view [**5**, p. 377] and is useful in the study of foliations *per se*, one wonders whether it is really necessary for the groupoid and hence the C^*-algebra. We put the question "why holonomy" to Georges Skandalis and he replied that one is "forced" to consider the concept when one tries to make the graph (of the equivalence relation defined by the leaves) into a manifold. After thinking about this for some time we came up with a theorem which justifies his remark. We hasten to add that Georges Skandalis also has a theorem justifying his remark (private communication) which predates ours—the private communication tarried on the desk of an intermediary colleague for some months during which time we had begun the fomulation of our own theorem. Of course, the two theorems are similar but not identical. The main point of our theorem is to show how the holonomy groupoid fits naturally between two groupoids canonically associated with the foliation, namely, the equivalence relation and the homotopy groupoid (in Skandalis' theorem, no mention is made of the homotopy groupoid). As byproducts of this investigation

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