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WHEN IS A FUZZY TOPOLOGY TOPOLOGICAL?

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ABSTRACT. In a 1981 paper R. Lowen defined topological fuzzy topologies (intuitively, fuzzy topologies that are not significantly fuzzy). He also proved that compact, Hausdorff fuzzy topologies must be topological. In this note Lowen's concept is described using semi-closure operators and the hypergraph functor. The two main results are:

1) The α-closure operators defined by a topological fuzzy topology are all identical and must be closure operators.

2) A fuzzy topology is topological if and only if the image under the hypergraph functor is a certain topological product.

Preliminaries. In this note the underlying lattice will be linearly ordered, complete, and completely distributive. The topology τ_r consists of L and all half-open intervals of the form $(\alpha, 1]$. A map into L is lower semi-continuous (lsc) if and only if it is continuous using τ_r on L. The Lowen definition [4] of fuzzy topology will be used: a family of fuzzy sets containing all constant maps and closed under arbitrary suprema and finite infima. If τ is a topology for X, $\omega(\tau)$ is the L-fuzzy topology consisting of all lsc maps from X into L. If T is an L-fuzzy topology for X, then $\iota(T)$ is the weakest topology that makes all elements of T lower semi-continuous. T is topological [5] provided $\omega(\iota(T)) = T$.

I. Relation to α-closure. For the definition of α-closure operators and their basic properties see [2] and [3]. The first lemma is easy to do directly and is implicit in [3].

LEMMA I.1. *Let (X, T) be an L-fts with* 0*-closure k_0. Then k_0 is a closure operator.*

PROPOSITION I.2. *Let (X, τ) be a topological space with closure c. For α in $L-\{1\}$, let k_α denote the α-closure operator induced by $\omega(\tau)$. Then $k_\alpha = c$.*

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