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DERIVATIONS FROM SUBALGEBRAS OF C^*-ALGEBRAS WITH CONTINUOUS TRACE

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1. Introduction. Let A be a C^*-algebra, $M(A)$ its multiplier algebra, B a C^*-subalgebra of A. Suppose $\delta: B \to A$ is a derivation of B into A, i.e., a linear map for which $\delta(ab) = a\delta(b) + \delta(a)b$, for all $a, b \in B$. Derivations of this type are most easily obtained by choosing an element m of $M(A)$ and setting $\delta(b) = mb - bm$, $b \in B$. Such derivations are said to be inner in $M(A)$, with generator m. In this paper, we begin the investigation of C^*-algebras A with the following property: for each C^*-subalgebra B of A and each derivation $\delta: B \to A$, there is an element $m \in M(A)$ for which $\delta(b) = mb - bm$, for all $b \in B$. We will say that a C^*-algebra with this property is hereditarily cohomologically trivial (HCT for short).

The HCT C^*-algebras are of interest for a number of reasons. The problem of studying them was first raised (without the terminology just introduced) by Kaplansky on p. 7 of [**11**], who was motivated by Sakai's famous theorem [**16**] that all derivations of simple, unital C^*-algebras are inner, and the fact that any derivation of a semisimple subalgebra into a central simple algebra can be extended to an inner derivations of the larger algebra. E. Christensen in [**4**, **5**] has attacked the difficult and as yet still unanswered question of whether $B(H)$, the algebra of all bounded operators on a Hilbert space H, is HCT, and has shown [**4**, Section 5] that all finite von Neumann algebras have this property. Akemann and Johnson have pointed out in the introduction to [**2**] the importance of investigating those pairs (B, A) of C^*-algebras A and C^*-subalgebras B of A for which every derivation of B into A is inner in $M(A)$; since the HCT C^*-algebras have this property for all C^*-subalgebras, a knowledge of them will be very useful in coming to grips with this more general problem of Akemann and Johnson (indeed, in the remarks which end [**13**], the present authors indicate at least one instance in which such a payoff actually occurs). Finally, J. Cuntz (private communication) has pointed out that knowledge of the structure of derivations of a C^*-subalgebra B into a C^*-algebra A would be useful in the study of the non-commutative K-theory of Kasparov [**12**]. In particular, the very nice

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