

THE EQUICONTINUOUS STRUCTURE RELATION AND EXTENSION OF CONTINUOUS EQUIVARIANT FUNCTIONS

JAN DE VRIES

ABSTRACT. In this paper we study injective objects in the category of all compact Hausdorff G -spaces, using methods from topological dynamics. In particular, we consider the question of when the equicontinuous structure relation of a subflow is the restriction of the equicontinuous structure relation of the full flow. Some necessary and sufficient conditions are given, one in terms of almost periodic functions on the flow, and another in terms of injective objects in the category of all compact Hausdorff G -spaces.

1. Introduction. This paper is in the borderline of general topology and topological dynamics. To be more precise: we use methods from topological dynamics to study a problem which arose in “equivariant topology”. By “equivariant topology” we mean the topological study of the category \mathcal{TOP}^G of all topological transformation groups with a fixed acting group G (G -spaces) and continuous equivariant mappings. “Topological study”, for the stress is not on the categorical aspects of this category (as for example in [18]), but on the topological ones. Roughly speaking, one considers a theorem in topology and then one examines the analogous situation in \mathcal{TOP}^G . This idea has been used in algebraic topology for some time, see e.g. [25, 26, and 27], to mention but a few references. As to equivariant general topology, see e.g. [20] or, for a survey of work by Yu.M. Smirnov and his co-workers, [24].

The present paper is devoted to the study of injective objects in the category of all (compact) Hausdorff G -spaces (roughly, try to find an analog for extension theorems like those of Dugundji, Borsuk and Arens). The problem is to find a non-trivial compact Hausdorff G -space which is injective for (or, as we shall also say, which is an extensor for) the class of all closed equivariant embeddings in the category of all compact Hausdorff G -spaces. If the topological group G is discrete or compact, then

1980 *Mathematics Subject Classification*: 54H15, 54C20, 54H20.

Key words: G -space, equicontinuous structure relation, extensor, injective object, compact convex set, almost periodic function on a flow.

Received by the editors on March 28, 1984 and in revised form on July 5, 1985.

Copyright © 1986 Rocky Mountain Mathematics Consortium