ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 4, Fall 1986

PARA-UNIFORMITIES, PARA-PROXIMITIES, AND H-CLOSED EXTENSIONS

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ABSTRACT. A generalized uniformity, called a para-uniformity, and its induced generalized proximity, called a para-proximity, are introduced and applied to the investigation of H-closed spaces and H-closed extensions of Hausdorff spaces.

H-closed spaces are characterized in terms of these structures, and the H-closed extensions of a Hausdorff space are characterized in terms of extensions of these structures. Moreover, collections of para-uniformities called superstructures are used to obtain all strict H-closed extensions of a non-H-closed Hausdorff space. Thus, the S-equivalence classes of H-closed extensions are described by a method similar to that of Fedorčuk for describing the R-equivalence classes.

0. Introduction. Alexandroff [1] remarked in 1960 that no method of systematically determining the H-closed extensions of a Hausdorff space had been found. In classifying (the isomorphism classes of) such extensions, the introduction of two equivalence relations discussed in [18] is helpful. We declare two H-closed extensions of a given space to be R-equivalent if they are θ-isomorphic and to be S-equivalent if their corresponding strict (or simple) extensions are isomorphic. In attempts to answer Alexandroff's remark, various authors have sought methods for obtaining all isomorphism classes, all R-equivalence classes, or all S-equivalence classes. (See, for instance, [2, 4, 7, 10, 11, 17 or 21].)

Fedorčuk [7] refers to the particular problem of constructing the H-closed extensions of a given Hausdorff space by means of uniformity or proximity-like structures as "Tychonoff's problem." He [7], Porter and Votaw [18] have shown that in general there are not enough such structures on a set to yield all isomorphism classes of either semiregular H-closed extensions or strict H-closed extensions of one of its Hausdorff topologies. According to results in [18] this implies that neither the R-equivalence classes nor the S-equivalence classes can be obtained in this manner, and

AMS *subject classification*: Primary 54D25, 54E05, 54E15; Secondary 54D30.
Key words and phrases: para-uniformity, para-proximity, H-closed extension.
Received by the editors on April 2, 1984 and in revised form on June 1, 1985.