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THE APPROXIMATE SOLUTION OF MONOTONE NONLINEAR OPERATOR EQUATIONS

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ABSTRACT. This paper is concerned with nonlinear equations involving monotone operators and compact perturbations of monotone operators. Projection methods determine approximate solutions. Such equations are put into the more general framework of regular operator approximation theory, which yields the convergence of approximate solutions under minimal hypothesis. Nonlinear integral equations of Urysohn type illustrate the theory.

1. Introduction and summary. There is a considerable literature on monotone nonlinear operator equations and compact perturbations of such equations. Principal applications are given by nonlinear integral equations of Urysohn and Hammerstein type. Some pertinent references are [5], [6], and [8].

Regular operator approximation theory [1], which is based on inverse-compactness concepts, provides a convenient general framework for the convergence of approximate solutions. The existence of solutions then follows in a natural way.

The gist of regular operator approximation theory is as follows. Let X and Y be Banach spaces. Let $A, A_n: X \to Y$, for $n = 1, 2, \ldots$. We shall compare equations

$$Ax = y, \qquad A_n x_n = y_n, \qquad x, x_n \in X, \qquad y, y_n \in Y.$$

Regular convergence $A_n \xrightarrow{r} A$ is a composite property. It includes continuous convergence,

$$A_n \xrightarrow{c} A: x_n \to x \Rightarrow A_n x_n \to Ax.$$

It also includes asymptotic regularity: if $\{x_n\}$ is bounded and $A_n x_n \to y$ on a subsequence, then $\{x_n\}$ has a convergent subsequence. Assume that $A_n \xrightarrow{r} A$, $y_n \to y$, and $\gamma > 0$. Define

$$S = \{x \in X: Ax = y, \|x\| \leqq \gamma\},$$
$$S_n = \{x_n \in X: A_n x_n = y_n, \|x_n\| \leqq \gamma\}.$$

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