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OPTIMAL ALGORITHMS FOR LINEAR PROBLEMS WITH GAUSSIAN MEASURES

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ABSTRACT. We study optimal algorithms for linear problems in two settings: the average case and the probabilistic case settings. We assume that the probability measure is Gaussian. This assumption enables us to consider a general class of error criteria. We prove that in both settings adaption does not help and that a translated spline algorithm is optimal. We also derive optimal information under some additional assumptions concerning the error criterion.

1. Introduction. In this paper we study the optimal reduction of uncertainty for linear problems in two settings: the average case setting and the probabilistic case setting.

By a linear problem we mean the problem of approximating Sf, where S is a linear operator defined on a separable Hilbert space F_1, when only partial information Nf on f is available. This partial information causes uncertainty. In the average case setting the intrinsic uncertainty is measured by the average size of the error of the best possible algorithm that uses N. In the probabilistic case setting it is measured by the probabilty that the error of the best possible algorithm is small. In this paper we assume that the probability measure on the space F_1 is Gaussian and the difference between Sf and x, the value given by an algorithm, is measured by $E(Sf\text{-}x)$, where E is an arbitrary error functional.

The average case setting has been studied in **[5, 7, 8]** for a rather general class of probability measures, assuming however that the error functional is of a special case. Typically it is assumed that $E(Sf-x) = \|Sf-x\|^2$ and $S(F_1)$ is a separable Hilbert space. Here, restricting the class of probability measures to Gaussian measures, we relax the assumption concerning the problem and the form of the error functional E. We are able also to study the probabilistic case setting.

The following results are obtained for both the average case and the probabilistic case settings:

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