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THE IDEAL STRUCTURE OF THE SPACE OF κ-UNIFORM ULTRAFILTERS ON A DISCRETE SEMIGROUP

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1. Introduction. Throughout this paper $(S, +)$ will denote an infinite discrete semigroup. It is well known that the operation $+$ on S extends uniquely to βS, the Stone-Čech compactification of S, so that $(\beta S, +)$ is a left-topological semigroup with S contained in its topological center. (By left-topological we mean that, for each $p \in \beta S$, the function λ_p, defined by $\lambda_p(q) = p + q$, is continuous. The topological center is the set of points at which also ρ_p is continuous, where $\rho_p(q) = q + p$. See **[2]** for an elementary derivation of this extension.)

Since βS is the maximal left-topological compactification of S **[2**, Theorem 2.4], its algebraic structure is of inherent interest. Each compact left-topological semigroup has a smallest two-sided ideal (called, for obscure historical reasons, the minimal ideal) which is the union of all of the minimal right ideals and is also the union of all of the minimal left ideals, **[3**, Theorem II. 2.2]. It is this ideal structure with which we are primarily concerned in this paper.

In an earlier paper **[9]** we characterized the minimal right ideals and minimal ideals of $(\beta N, +)$ and $(\beta N, \cdot)$. It was observed later that the same results held for any discrete semigroup. We were led by this observation to consider the extent to which these and other earlier results extended to certain natural subsemigroups of βS.

The points of βS are the ultrafilters on S, each point $x \in S$ being identified with the principal ultrafilter $\hat{x} = \{A \subseteq S: x \in A\}$. For $A \subseteq S$, we let $\bar{A} = \{p \in \beta S: A \in p\}$. The set $\{\bar{A}: A \subseteq S\}$ forms a basis for the open sets of βS (as well as a basis for the closed sets). See **[5]** or **[7]** for a detailed construction of βS as a space of ultrafilters.

Associated with each ultrafilter p is a cardinal, $\|p\|$, called the *norm* of p. It is the minimum of $\{|A|: A \in p\}$. An ultrafilter p with $\|p\| \geqq \kappa$ is called κ-uniform. The space $U_\kappa(S)$ of κ-uniform ultrafilters on S is closed in βS and, as we shall see in §2, is very often a subsemigroup of βS. (For

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