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A COMPLETE CHARACTERIZATION OF THE LEVEL SPACES OF R (I) AND I (I)

R. LOWEN

ABSTRACT. By means of T_0-modifications we are able to precisely describe all level spaces of $\mathbf{R}$ (I) and I (I) and to show that there are only 3 nonhomeomorphic level spaces of $\mathbf{R}$ (I) and only 4 nonhomeomorphic level spaces of I (I). A large list of α-properties of both $\mathbf{R}(I)$ and $I(I)$ is deduced, and an open problem with regard to α-compactness is solved.

1. Preliminaries. I denotes the unit interval and $I_1 = [0, 1[$. If X is a topological space and $x \in X$ we denote its neighborhoodfilter $\mathcal{N}(x)$.

We recall that a topological space is called hyperconnected (resp. ultraconnected) if no disjoint open (resp. closed) sets exist [**12**].

If $\mathscr{B}$ is a filterbase then the generated filter is denoted $[\mathscr{B}]$. If (X, Δ) is a fuzzy topological space then for any $\alpha \in I_1$ the α-level space, denoted $\iota_\alpha(X)$, is the topological space $(X, \iota_\alpha(\Delta))$ where $\iota_\alpha(\Delta) = \{\mu^{-1}]\ \alpha, 1] \mid \mu \in \Delta\}$ (see [**5**], [**6**]). We use the simplified version of $\mathbf{R}$ (I) and I (I) introduced in [7]. That means that throughout this paper $\mathbf{R}$ (I) is the set of all non-increasing left continuous maps from $\mathbf{R}$ to I with supremum equal 1 and infimum equal 0.

The fuzzy topology considered on this set is determined by the subbasis $\{L_x, R_y \mid x, y \in \mathbf{R}\}$ where L_x and R_y are defined as

$$L_x(\lambda) = 1 - \lambda(x)$$
$$R_y(\lambda) = \lambda(y+)$$

for any $\lambda \in \mathbf{R}$ (L).

We also recall that I (I) is the subspace of $\mathbf{R}$ (I) defined by $\mu \in I$ (I) if and only if $\mu(0) = 1$ and $\mu(t) = 0$ for all $t > 1$. For more information on these spaces see [**1**], [**2**], [**3**], [7], [**9**] and [**10**]. In [**10**] the numbers $a(\mu, \alpha)$, $b(\mu, \alpha)$, $a^*(\mu, \alpha)$, $b^*(\mu, \alpha)$ for any $\mu \in \mathbf{R}(I)$ and $\alpha \in I$ were introduced. In [7] we showed the following proposition which we require in the sequel.

PROPOSITION 1.1. *For any* $\mu \in \mathbf{R}$ (I) *and* $\alpha \in I$
(i) $a(\mu, \alpha) = \inf \mu^{-1}[0, 1 - \alpha[$,

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