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APPROXIMATION OF LINEAR OPERATORS ON A WIENER SPACE

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ABSTRACT. We study optimal algorithms and optimal information in an average case model for linear problems in a Wiener space. We show that a linear algorithm is optimal among all algorithms. We illustrate the theory by interpolation, integration and approximation. We prove that adaption does not help.

1. Introduction. In a series of pioneering papers commencing with [**4**], Larkin studied average case error, mostly for linear problems in a Hilbert space equipped with a Gaussian measure. The average case model was further developed in [**8**], [**13**], and [**14**].

Following the average case model of [**13**], in this paper we study linear problems in a Wiener space. A Wiener space is a Banach space of continuous functions equipped with a Wiener measure. Linear problems in a Wiener space were first studied in [7], where optimality was considered in the class of linear algorithms. This paper investigates optimality in the class of all algorithms. It also studies optimal information and adaptive information.

We summarize the main contents of this paper.

In §3 we formulate the problem and recall the concepts of information, algorithm, radius of information, optimal information and optimal algorithm.

We address the problem of interpretation in §4, and we derive the optimal algorithm, which turns out to be linear, and the radius of information.

Based on the results in §4, we study the problem of approximation of continuous linear functionals in §5. We derive the optimal algorithm and the radius of information. As a specific case, we investigate the problem of integration.

In §6 we study the problem of approximation of bounded linear operators. As a specific case we study the approximation problem.

In §7 we discuss adaptive information versus nonadaptive information,

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