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LOCAL FACTORS OF FINITELY GENERATED WITT RINGS

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ABSTRACT. The Witt rings considered here are the abstract Witt rings in the sense of Marshall [3]. A local Witt ring is one with a unique non-trivial 2-fold Pfister form. Our main result gives necessary and sufficient conditions for a finitely generated Witt ring to be a product (in the category of Witt rings) of two Witt rings, one of which is local. The basic motivation is to develop a tool for the study of whether every finitely generated Witt ring is of elementary type (that is, can be built from local Witt rings $\mathbf{Z}/4\mathbf{Z}$ and $\mathbf{Z}/2\mathbf{Z}$ by a succession of products and group ring extensions), cf. [3; problem 4, p. 123].

1. Introduction. R will always denote a non-degenerate finitely generated Witt ring and G will be the multiplicative subgroup of one-dimensional forms in R. The category of Witt rings is equivalent to the category of quaternionic structures and also to that of the quaternionic schemes defined in [1]. We let q denote the quaternionic mapping associated with R. For $a \in G$, $D\langle 1, a\rangle = \{b \in G | q(b, -a) = 0\}$ is the value set of the form $\langle 1, a\rangle$; $i(a)$ will denote the index of $D\langle 1, a\rangle$ in G. For a subset K of G, we let $Q(K) = \{q(k, x) \mid k \in K, x \in G\}$. If $K = \{k\}$, we write $Q(k)$ for $Q(K)$. We will be mainly concerned with the existence of elements $a \in G$ such that $i(a) = 2$, equivalently, such that $|Q(-a)| = 2$.

For Witt rings R_1 and R_2 we let $R_1 \times_w R_2$ denote the product of R_1 and R_2 in the category of Witt rings. We say R_1 is a local factor of R if $R \cong R_1 \times_w R_2$ with R_1 a local Witt ring. C_2 denotes the group of order 2 and $R[C_2]$ denotes the group ring of C_2 with coefficients in R. Details on products and group rings of Witt rings may be found in [3].

For $a \in G$, we let $M(a) = \{m \in G | i(m) = 2, i(-am) = 2 \text{ and } D\langle 1, m\rangle \neq D\langle 1, a\rangle\} \cup \{a\}$, and we let $H(a) = \bigcap_{m \in M(a)} D\langle 1, m\rangle$. We say a is a local element if $i(a) = 2$ and $\rho \notin Q(H(a))$, where ρ is the unique non-trivial element in $Q(-a)$. The main goal of this paper is to prove the following

THEOREM 1.1. *Let R be a finitely generated non-degenerate Witt ring. R has a local factor if and only if R has a local element.*

We take a moment here to motivate our definition of local element.

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