ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 3, Summer 1986

α-SEPARATION AXIOMS AND α-COMPACTNESS IN FUZZY TOPOLOGICAL SPACES

A.S. MASHHOUR, M.H. GHANIM, M.A. FATH ALLA

ABSTRACT. In **[11]** Rodabaugh introduced the concept of α-Hausdorff fuzzy topological spaces which is compatible with α-compactness **[4]** and fuzzy continuity. It is the purpose of this paper to extend these concepts. We define and study $\alpha - T_i$ ($i = 0, 3, 4$), $\alpha - T_i'$ ($i = 0, 1, 2, 3, 4$), α-almost compact and α-nearly compact fuzzy topological spaces. Also, we define α-continuous mappings as a generalization of F-continuous mappings. Finally, we define αS-closed fuzzy spaces and study some of their properties.

1. Preliminaries. Let X be a set. If $A \subset X$, $\mu(A)$ will denote the characteristic function for A defined on X into the unit interval $I = [0, 1]$. A fuzzy topology τ on X is a family of fuzzy sets (functions from X into I) which is closed under arbitrary suprema and finite infima and which contains $0 = \mu(\phi)$ and $1 = \mu(X)$. A pair (X, τ), where τ is a fuzzy topology on X, is called a fuzzy topological space (abbreviated as fts). A fuzzy set u of an fts (X, τ) is regular open (resp. regular closed) if $u = \bar{u}^0$ (resp. $u = \bar{u}^0$), it is fuzzy semiopen if $u \leqq \bar{u}^0$. For notion and results used but not defined or shown we refer to **[3, 5, 13, 16, 18]**.

DEFINITION 1.1 **[11]**. Let (X, τ) be an fts and $A \subset X$. A point $x \in X$ is an α (resp. α^*)-cluster point of A if for each $u \in \tau$ with $u(x) > \alpha$ (resp. $u(x) \geqq \alpha$), $u \wedge \mu(X/A) \neq 0$, where $\alpha < 1$ (resp. $\alpha < 0$). The family of all α (resp. α^*)-cluster points of A will be denoted by A^α(resp. A^{α^*}). The α (resp. α^*) closure of A is the union of A and its α (resp. α^*) cluster points and will be denoted by $\mathrm{Cl}_\alpha(A)$ (resp. $\mathrm{CL}_{\alpha^*}(A)$). The subset A of X is α (resp. α^*)-closed if $\mathrm{Cl}_\alpha(A) \subset A$ (resp. $\mathrm{Cl}_{\alpha^*}(A) \subset A$).

PROPOSITION 1.2 [11]. *Let (X, τ) be an fts. Then*
(i) *a subset A of X is α (resp. α^*)-closed if and only if for each point $x \in X \backslash A$ there is $u \in T$ such that $u(x) > \alpha$ (resp. $u(x) \geqq \alpha$) and $u \wedge \mu(A) = 0$.*
(ii) *arbitrary intersection of α (resp. α^*)-closed sets is α (resp. α^*)-closed,*
(iii) *a finite union of α (resp. α^*)-closed sets is α (resp. α^*)-closed, and,*
(iv) *the inverse image of each α (resp. α^*)-closed set under an F-continuous mapping is α (resp. α^*)-closed.*

Received by the editor on October 1, 1983, and in revised form on November 17, 1984.