ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 3, Summer 1986

ON NEIGHBOURHOODS OF UNIVALENT CONVEX FUNCTIONS

RICHARD FOURNIER

Introduction. Let A denote the class of analytic functions f in the unit disk $E = \{z \mid |z| < 1\}$ with $f(0) = f'(0) - 1 = 0$. For $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ in A and $\delta \geqq 0$ Ruscheweyh has defined the δ-neighbourhood $N_\delta(f)$ as follows:

$$N_\delta(f) = \{g \in A \mid g(z) = z + \sum_{k=2}^{\infty} b_k z^k \text{ and} \sum_{k=2}^{\infty} k|a_k - b_k| \leqq \delta\}.$$

He has shown in [3], among other results, that if $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k \in C$, then

$$N_{d_n}(f) \subset S^* \text{ if } d_n = 2^{-2/n} \tag{1}$$

where $C(S^*)$ denotes the class of normalized convex (starlike) univalent functions in A. Ruscheweyh also asked in [3] if results analogous to (1) would hold if the class C were replaced by some of its subclasses.

Let $t > 1/2$. We consider the following subclasses of A:

$$(S^*)_t = \{f \in A \mid \left|\frac{zf'(z)}{f(z)} - t\right| < t, z \in E\}$$

and

$$(C)_t = \{f \in A \mid \left|\frac{zf''(z)}{f'(z)} + 1 - t\right| < t, z \in E\}.$$

It is clear that $(S^*)_t \subset S^*$ and $(C)_t \subset C$. The classes $(S^*)_t$ and $(C)_t$ have been studied by several authors (see for example [4], [5], [6]). We prove

THEOREM 1. *Let $t \geqq 1$ and $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k \in (C)_t$. Then $N_{\delta_n}(f) \subset (S^*)_t$ if $\delta_n = (2 - 1/t)^{-(1/n)(2-1/t)/(1-1/t)}$. The value given to δ_n is the best possible.*

THEOREM 2. *Let $1/2 < t \leqq 2$ and $f(z) = z + \sum_{k=2}^{\infty} a_k z^k \in (C)_t$. Then $N_\delta(f) \subset (S^*)_t$ if $\delta = \inf_{z \in E} |t(f(z)/z)| - |f'(z) - t(f(z)/z)|$.*

THEOREM 3. *Let $1/2 < t \leqq 1$ and $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k \in (C)_t$. Then*

Received by the edictor on November 13, 1984.